Holidays' Homework, 2024-25

Class- XII

Mathematics

Chapter-1 Relation and Functions

Q.1 Consider f:R – $\left\{-\frac{4}{3}\right\} \rightarrow R - \left\{\frac{4}{3}\right\}$ given by f(x) =$\frac{4x+3}{3x+4}$. SHow that f is bijective function.

Q.2 Show that the function f: R → R defined by f(x) = $\frac{x}{x^2+1}$ $\forall x \in R$ is neither one-one nor on – to.

Q.3 Let A = R – {3} , B = R – {1}. Let f : A → B be defined by f(x) = $\frac{x-2}{x-3}$ ∀ x $\neq 3$. Show that f is one-one & on-to Function.

Q.4 Consider f : $R_+ \rightarrow [-5, \propto)$ given by f(x) = $9x^2 + 6x - 5$, show that f is bijective, when R_+ is the set of all non-negative real numbers.

Q.5 A function f : R → (-1,1) is defined by f(x) = $\frac{x}{1+|x|}$ $\forall x \in R$, then prove that f is one-one and on – to

Q.6 f : N → N be defined by f(x) = $\begin{cases} \frac{n+1}{2} & if\ n\ is\ odd \\ \frac{n}{2} & if\ n\ is\ even\ , \end{cases}$ $\forall n \in N$, show that f is not one-one but on to function.

Q.7 Prove that the function f : N → N given by f(x) = $x^2 + x + 1$ is one-one but not on-to

Q.8 Show that the function f : R → R given by f(x) = $x^3 + x$ is a bijective function.

Q.9 Show that the function f : N → N defined by f(x) = n-$(-1)^n$ $\forall n \in N$ is a bijection.

Q.10 Show that f : N → N given by f (x) = $\begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$, is both one-one and on-to

Q.11 Determine whether the relation R on Z defined by (a,b) ∈ R ⇔ $|a-b| \leq 5$ is reflexive, symmetric and transitive

Q.12 Let a relation R_1 on the set R of real numbers be defined as (a,b) ∈ $R_1 \Leftrightarrow 1 + ab > 0\ \forall\ a, b \in R$. Show that R_1 is reflexive and symmetric but not transitive.

Q.13 Check whether the relation R in set of real numbers R defined by R = { (a,b) : a≤ b^3} is reflexive, symmetric or transitive.

Q.14 Show that the relation R on the set A = {x ∈ z:0 ≤ x ≤ 12} given by R = {(a,b) : $|a-b|$ is multiple of 4 } is an equivalence relation. Also find the set of all elements related to 2.

Q.15 Show that the relation R in the set A = {1,2,3,4,5} given by R = {(a,b) : $|a-b|$is even}, is an equivalence relation. Also show that all the elements of {1,3,5} are related to each other and all the element of {2,4} are related to each other. But no element of {1,3,5} is related to any element of {2,4}

Q.16 Let A = {1,2,3……………………..,9} and R be the relation in A x A defined by (a,b) R (c,d) if a + d = b + c for (a,b) (c,d) ∈ A x A Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)]

Q.17 Let N denote the set of all natural numbers and R be the relation on N x N defined by (a,b) R (c,d) ⇔ ad (b+c) = bc (a+d). Show that R is an equivalence relation on N x N

Q.18 If R_1 and R_2 are two equivalence relations on a set A, then show that $R_1 \cap R_2$ is also an equivalence relation on A.

Q.19 Let R be a relation on the set A of ordered pairs of Positive integers defined by (x,y) R (u, v) if and only if xv – yu. Show that R is an equivalence relation.

Q.20 Determine whether the relation R defined on the set R of all real numbers as R = {(a,b) : a, b ∈ R and a – b + $\sqrt{3}$ ∈ S} where S is the set of all irrational numbers, is reflexive symmetric and transitive.

Chapter-2

Inverse Trigonometric Functions

Q.1 Find Domain of the following functions:

(a) $f(x) = \sin^{-1}(2x-5)$ (d) $f(x) = \sin^{-1}(-x^2)$

(b) $f(x) = \sin^{-1}\sqrt{x-1}$ (e) $f(x) = \cos^{-1}(4-x^2)$

(c) $f(x) = \cos^{-1}(2x-1)$ (f) $f(x) = \sin^{-1}x + \cos x$

Q.2 Evaluate $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$

Q.3 Evaluate $\sin^{-1}[\cos\left\{\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$

Q.4 Evaluate : $\cos\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right]$

Q.5 Evaluate : $\sin\left(2\tan^{-1}\frac{1}{4}\right) + \cos\left(\tan^{-1}2\sqrt{2}\right)$

Q.6 Evaluate : $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$

Q.7 If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$, *then find* x

Q.8 Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2 \quad -1 < x < 1$

Q.9 Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{2}\right)$

Q.10 Simplify : $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$ for $x < -1$

Q.11 Write the following functions in the simplest form:

(a) $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), \frac{-1}{\sqrt{3}} < \frac{x}{a} < \frac{1}{\sqrt{3}}$

(b) $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

(c) $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, \frac{-\pi}{2} < x < \frac{\pi}{2}$

Q.12 Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$

Q.13 Solve for x : $\cos(2\sin^{-1}x) = \frac{\pi}{2}$

Q.14 Solve the following equations:

(a) $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

(b) $\cos_{-1}(\sqrt{3}x) + \cos^{-1}x = \frac{\pi}{2}$

Q.15 Find least value of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$.

Q.16 If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, find x

Q.17 If $x,y,z \in [-1]$ such that $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$ then find the values of

(a) $xy + yz + zx$ (b) $x(y+z) + y(z+x) + z(x+y)$

Q.18 If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ then find the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$

Q.19 Prove that $\sec^2(\tan^{-1}z) + \text{cosec}^2(\cot^{-1}3) = 15$

Q.20 If $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$ and $\tan^{-1}x - \tan^{-1}y = 0$, then find the value of $x^2 + xy + y^2$

Chapter-3 & 4

Matrices & Determinants

Q.1 If $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix} + \begin{bmatrix}1 & y\\0 & 1\end{bmatrix} = \begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$, find (x-y)

Q.2 Find non zero value (s) of x satisfying the matrix equation:

$$x\begin{bmatrix}2x & 2\\3 & x\end{bmatrix} + 2\begin{bmatrix}8 & 5x\\4 & 4x\end{bmatrix} = \begin{bmatrix}x^2+8 & 24\\10 & 6x\end{bmatrix}$$

Q.3 Find matrix A such that $\begin{bmatrix}2 & -1\\1 & 0\\-3 & 4\end{bmatrix}A = \begin{bmatrix}-1 & -8\\1 & -2\\9 & 22\end{bmatrix}$

Q.4 Find the value of x such that $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\2 & 5 & 1\\15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\2\\x\end{bmatrix} = 0$

Q.5 Let $A = \begin{bmatrix}2 & -1\\3 & 4\end{bmatrix}, B = \begin{bmatrix}5 & 2\\7 & 4\end{bmatrix}, C = \begin{bmatrix}2 & 5\\3 & 8\end{bmatrix}$. Find matrix D such that CD-AB =0

Q.6 (i) If A is a square matrix such that $A^2 = A$, then prove that $(I + A)^3 - 7A = I$

(ii) If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A-I)^3 + (A + A)^3 - 7A$

Q.7 If $P(x) = \begin{bmatrix}\cos x & \sin x\\-sinx & \cos x\end{bmatrix}$, then show that P(x) P(y) = P (x + y)

Q.8 If $A\begin{bmatrix}0 & -tan\frac{\alpha}{2}\\tan\frac{\alpha}{2} & 0\end{bmatrix}$ and I is the unit matrix of order 2 x 2 ; show that $I + A = (I - A)\begin{bmatrix}\cos\alpha & -sin\alpha\\sin\alpha & \cos\alpha\end{bmatrix}$

Q.9 There are two families. Family P consists of 3 men, 3 women and 12 children . Family Q consists of 2 men and 3 women and 4 children. The recommended daily allowance for calories is men : 2400, women: 2000 , children : 1400 and for proteins men : 60g, women : 40 g and children : 35 g

(i) Using matrix multiplication , calculate the total requirement of calories and proteins of each of the two families.

(ii) Which family is an ideal family.

Q.10 If A = [3 5], B = [7 3], then find a non zero matrix C such that AC= BC.

Q.11 If AB= A and BA = B then show that $A^2 = A$ and $B^2 = B$

Q.12 If $A = \begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}$. Then show that A is a root of the polynomial $f(x) = x^3 - 6x^2 + 7x + 2$.

Q.13 Find the value of k if the area of the triangle is 35 sq.m. with the vertices (k, 4) , (2, -6) and (5,4).

Q.14 For the matrix $A= \begin{bmatrix}1 & 1 & 1\\1 & 2 & -3\\2 & -1 & 3\end{bmatrix}$ show that $A^3 - 6A^2 + 5A + 11I_3 = 0$. Hence, find A^{-1}

Q.15 If $A = \begin{bmatrix}2 & -3 & 5\\3 & 2 & -4\\1 & 1 & -2\end{bmatrix}$. Find A^{-1}. Use it to solve the system of equations 2x – 3y +5z =11, 3x + 2y – 4z = -5, x + y-2z =-3

Q.16 Use the product $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ to solve the system of equations x +3z =0, -x +2y -2z =4, 2x -3y +4z =-3

Q.17 The monthly income of Aryan and Babban are in the ratio 3:4 and their monthly expenditure are in the ratio 5:7. If each saves Rs. 15000 per month, find their monthly income using matrix method.

Q.18 Solve the system of equations using matrix method:

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4, \frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2, x, y, z \neq 0$

Q.19 Find a matrix X for which

$$\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$

Q.20 A total amount of Rs. 7000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and 8 ½ % respectively. The total annual interest from these three accounts is Rs. 550. Equal amounts have been deposited in the 5% and 8% saving accounts. Find the amount deposited in each of the three accounts with the help of matrices.

Chapter- 12

Linear Programming Problems

Q.1 Solve the following LPP graphically :

Maximise Z= 1000z + 600y

Subject to the constraints

$x + y \leq 200$

$x \geq 20$

$y - 4x \geq 0$

$x, y \geq 0$

Q.2 Solve the following LPP graphically:

Minimise Z = 5x + 10y

Subject to constraints

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

$x, y \geq 0$

MCQ's

Chapter-1

Q.1 If R = {(x,y) : x,y ∈ Z, $x^2 + y^2 \leq 4$} is a relation in set Z, then domain of R is :

(a) {0,1,2} (b) {-2, -1, 0,1,2} (c) {0,-1,-2} (d) {-1,0,1}

Q.2 Let A ={x,y,z} and B ={1,2} then the number of relations from A to B is:

(a) 32 (b) 64 (c) 128 (d) 8

Q.3 The relation R defined in A = {1,2,3} by a Rb, if $|a^2 - b^2| \leq 5$. Which of the following is false?

(a) R = {(1,1), (2,2), (3,3), (2,1), (1,2), (2,3), (3,2)}

(b) $R^{-1} = R$

(c) Domain of R = {1,2,3}

(d) Range of R = {5}

Q.4 Total number of equivalence relations defined in the set S = {a,b,c} is:

(a) 5 (b) 3! (c) 23 (d) 33

Q.5 Let A = {1,2,3}.Then number of equivalence relations containing (1,2) is:

(a) 1 (b) 2 (c) 3 (d) 4

Q.6 Let A = {1,2,3} and R = {(1,2), (2,3)} be a relation in A. Then , the minimum number of ordered pairs may be added, so that R becomes an equivalence relation is:

(a) 7 (b) 5 (c) 1 (d) 4

Q.7 For real numbers x and y, we write xRy $\Leftrightarrow$ x – y + $\sqrt{2}$ is an irrational number. Then the relation R is:

(a) reflexive (b) symmetric (c) transitive (d) None of these

Q.8 A function f : R $\rightarrow$ R defined by f(x) -2 + x^2 is:

(a) not one-one (b) one-one (c) not onto (d) neither one-one onto

Q.9 Let A = { x : $1 \leq x \leq 1$} and f: A $\rightarrow$ A is a function defined by f(x) = x $|x|$, then f is:

(a) a bijection (b) injection but not surjection

(c) surjection but not injection (d) neither injection nor surjection

Q.10 Range of f(x) = $\sin^{-1}$x + $\tan^{-1}$ x + $\sec^{-1}$ x is:

(a) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (b) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (c) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (d) None of these

Q.11 $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to:

(a) π (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Q.12 The value of sin $\left[tan^{-1}(-\sqrt{3}) + cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is

(a) 1 (b) -1 (c) 0 (d) None of these

Q.13 The value of $\sin^{-1}\left[sin\left(-\frac{17\pi}{8}\right)\right]$is:

(a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{8}$ (c) $-\frac{\pi}{8}$ (d) $\frac{\pi}{12}$

Q.14 The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is:

(a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{3}$ (c) $\frac{10\pi}{3}$ (d) 0

Q.15 Evaluate the determine $\Delta = \begin{vmatrix} log_3 512 & log_4 3 \\ log_3 8 & log_4 9 \end{vmatrix}$

(a) 15/2 (b) 12 (c) 14/3 (d) 6

Q.16 If A = $\begin{bmatrix} a & 4 \\ 4 & 0 \end{bmatrix}$ and $|A^3|$ = 729, then the value of 'a' is equal to:

(a) $\pm$ 6 (b) $\pm$ 3 (c) $\pm$ 4 (d) $\pm$ 5

Q.17 If A is 3 x 3 matrix such that $|A|$ =8, then $|3A|$ equals :

(a) 8 (b) 24 (c) 72 (d) 216

Q.18 If A and B are square matrices each of order 3 and $|A|$ =5, $|B|$ =3, then the value of $|3AB|$ is:

(a) 27 (b) 15 (c) 405 (d) 42

Q.19 If Aij denotes the cofactor of the element aij of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$ then value of

$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ is:

(a) 0 (b) 5 (c) 10 (d) -5

Q.20 If C_{ij} denotes the cofactor of element Pij of the matrix P = $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$ then the value of $C_{31}\,C_{23}\,(P_{31} - P_{23})$ is:

(a) 5 (b) 24 (c) -24 (d) -5

Q.21 If A is singular matrix, then A. (adj A) is:

(a) Singular (b) non-singular (c) symmetric (d) not defined

Q.22 Given that A is a square matrix of order 3 and $|A| = -4, then\ |adj\ A|$ is equal to:

(a) -4 (b) 4 (c) -16 (d) 16

Q.23 The number of all possible matrices of order 2 x 3 with each entry 1 or 2 is:

(a) 16 (b) 6 (c) 64 (d) 24

Q.24 If $\begin{bmatrix}4\\2\\3\end{bmatrix}\begin{bmatrix}1 & 3 & -3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = 0$, then x +3y – 3z is:

(a) 1 (b) 3 (c) 4 (d) 0

Q.25 If matrices A and B are of order 3 x n and m x 5 respectively, then the order of matrix C = 5A +3B is:

(a) 3 x 5 and m = n (b) 3 x 5 (c) 3 x 3 (d) 5 x 5

Q.26 If A = $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$, then A^{100} is equal to

(a) 2^{100} A (b) 2^{99} A (c) 100A (d) 299A

Q.27 In a linear programming problem, the constraints on the decision variables x and y are x – 3y $\geq$ o, y $\geq$0, 0$\leq$ x $\leq$ 3. The feasible region.

(a) is not in the first quadrant (b) is bounded in the first quadrant

(c) is unbounded in the first quadrant (d) does not exist

Q.28 A linear programming problem is as follows:

Minimise z = 30x + 50 y subject to the constraints,

3x + 5y $\geq$ 15

2x + 3y $\leq$ 18

X $\geq$ 0, y $\geq$ 0

In the feasible region the minimum value of z occurs at:

(a) A unique point (b) no point

(c) infinitely many points (d) two points only

Q.29 The corner points of the feasible region determined by a set of constraints (Linear inequalities) are P (0,5), Q (3,5), R (5,0) and S (4,1) and the objective function is Z = ax + 2by where a,b > 0. The condition on a and b such that the maximum Z occurs at Q and S is:

(a) a – 5b = 0 (b) a – 3b = 0 (c) a – 2b =0 (d) a – 4b = 0

Q.30 The graph of the following linear programming problem when the conditions are 50x +25y ≤ 500, $x + y \leq 12 \; and \; x \geq 0, y \geq o \; is$:

(a) feasible (b) unbounded (c) bounded (d) None of the above

Q.31 The graph of the inequality 3x + 4y < 12 is:

(a) half plane that contains the origin

(b) half plane that neither contains the origin nor the points of the line 3x +4y = 12.

(c) whole XOY-plane excluding the points on line 3x+4y = 12

(d) None of these

Activities: i) Activity -3 (Based on Functions)

Model – (i) One-one & on-to Functions. (Boys)

(ii) Inverse Trigonometric Functions. (Girls)

Chart – (i) Domain & Range of all inverse trigonometric Functions. (Boys)

(ii) Number of Functions, Number of one-one functions & on-to Functions. (Girls)